

ENUMERATION OF PERMUTATION CLASSES BY INFLATION OF INDEPENDENT SETS OF GRAPHS

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(This talk is based on joint work with C. Bean and H. Ulfarsson.)

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Bean, Tannock and Ulfarsson [1] show a link between the permutations in $\text{Av}(123)$ and $\text{Av}(132)$ to independent sets of certain graphs. We extend their results to enumerate $\text{Av}(2314, 3124)$, and moreover certain subclasses obtained by adding patterns of the form $1 \oplus \pi$ where π is skew-indecomposable. Extending the idea further allows us to get the enumeration of five classes and certain subclasses of these. Overall, this technique gives a unified way of enumerating a total of 48 classes avoiding patterns of length 4 and many more of longer length.

More precisely, we choose an independent set of size k in the graph U_n together with a list of k non-empty permutations in $\text{Av}(2314, 3124, P)$ where P is a set of skew-indecomposable permutations. We establish a bijection between these objects and permutations in $\text{Av}(2314, 3124, 1 \oplus P)$. From [1], we get the generating function, $F(x, y)$, where the coefficient of $x^n y^k$ gives the number of independent set of size k in U_n . We show that:

Theorem 1. *Let P be a set of skew-indecomposable permutations and $A(x)$ be the generating function of $\text{Av}(2314, 3124, P)$. The generating function of $\text{Av}(2314, 3124, 1 \oplus P)$ is $B(x) = F(x, A(x) - 1)$.*

This can be used to enumerate eight classes avoiding length 4 patterns, and many more avoiding longer patterns. Moreover, a similar theorem can be stated for the classes $\text{Av}(2413, 3142, 1 \oplus P)$ where all π in P are sum-indecomposable. This can be used to enumerate eight more classes avoiding length 4 patterns.

We then describe new graphs and provide a closed formula for the generating function counting independent sets. We show how these can be used to enumerate $\text{Av}(S, 1 \oplus P)$ where S is any subset of size 3 of $\{2314, 3124, 3142, 2413\}$ and P a set of indecomposable permutations (either skew or sum depending on the subset S chosen). This gives the enumeration of 32 new classes avoiding length 4 patterns.

- [1] Christian Bean, Murray Tannock and Henning Ulfarsson. *Pattern avoiding permutations and independent sets in graphs*. arXiv:1512.08155 (2015). Submitted.