

RESTRICTED ROOTED NON- SEPARABLE PLANAR MAPS

Preliminary Report

Henning Úlfarsson, Reykjavik University

2012 Joint Mathematics Meetings

Sergey Kitaev



Pavel Salimov



- Joint work with

Christopher Severs

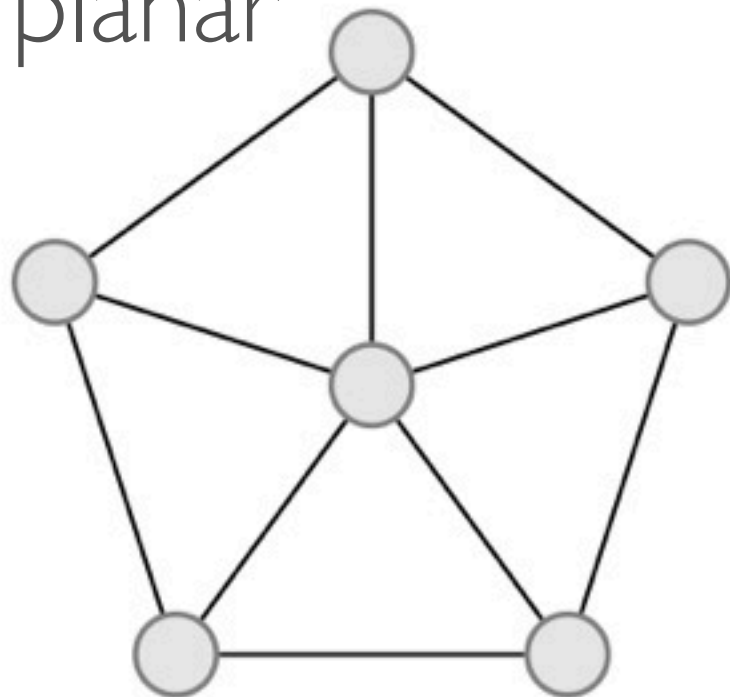


- Main idea: Translation of properties between
 - planar maps
 - trees
 - permutations

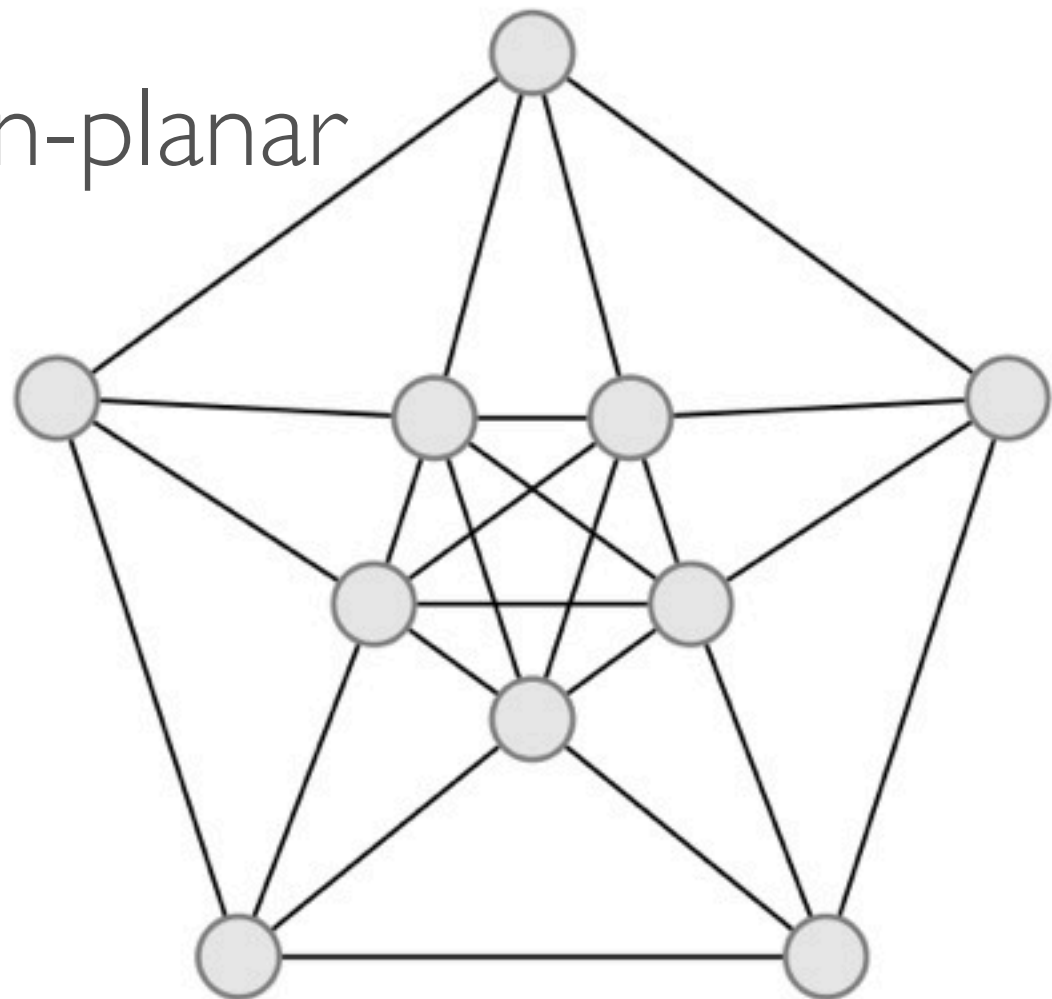
PLANAR MAPS

- A *planar map* is a graph drawn in the plane without edges crossing

planar

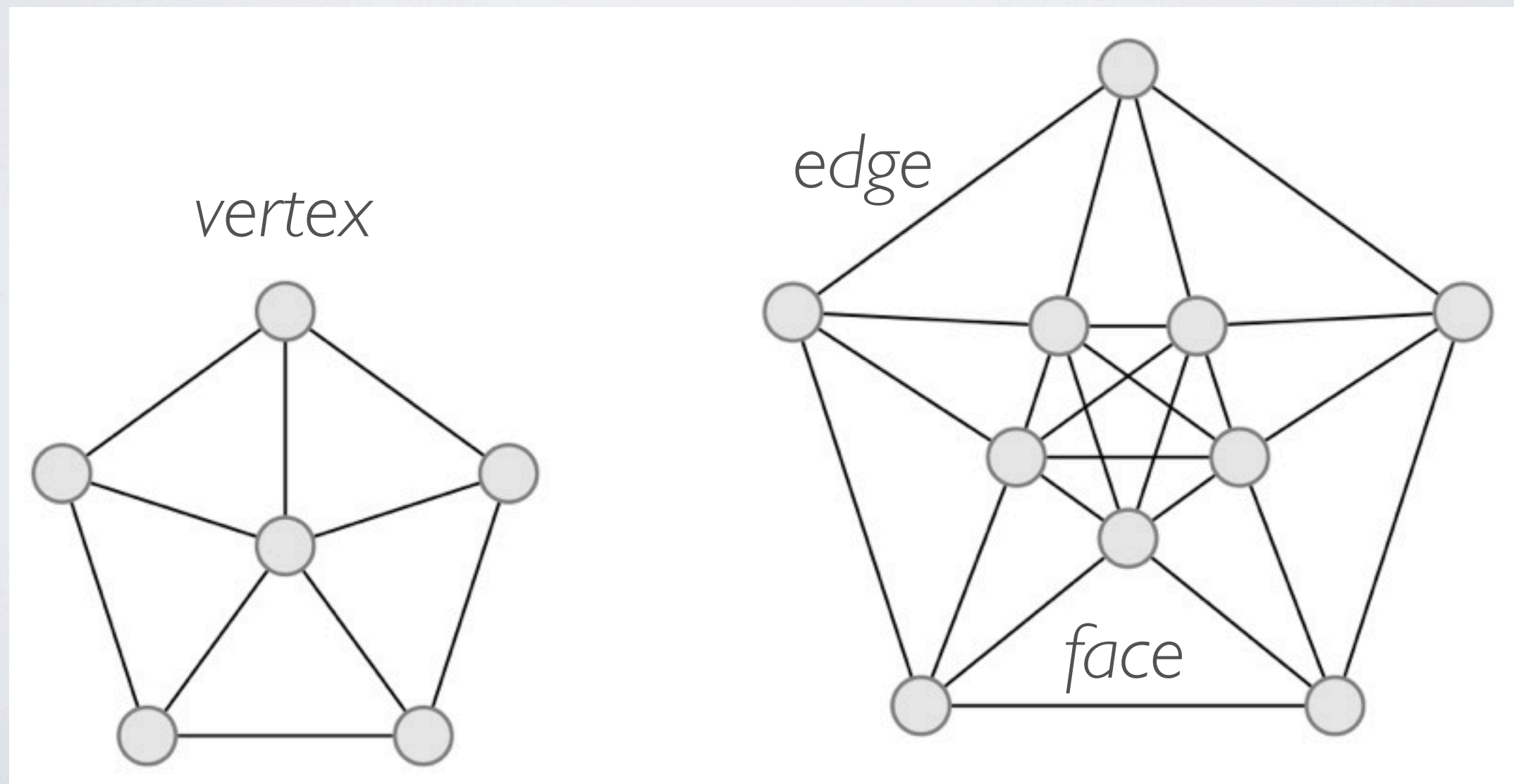


non-planar



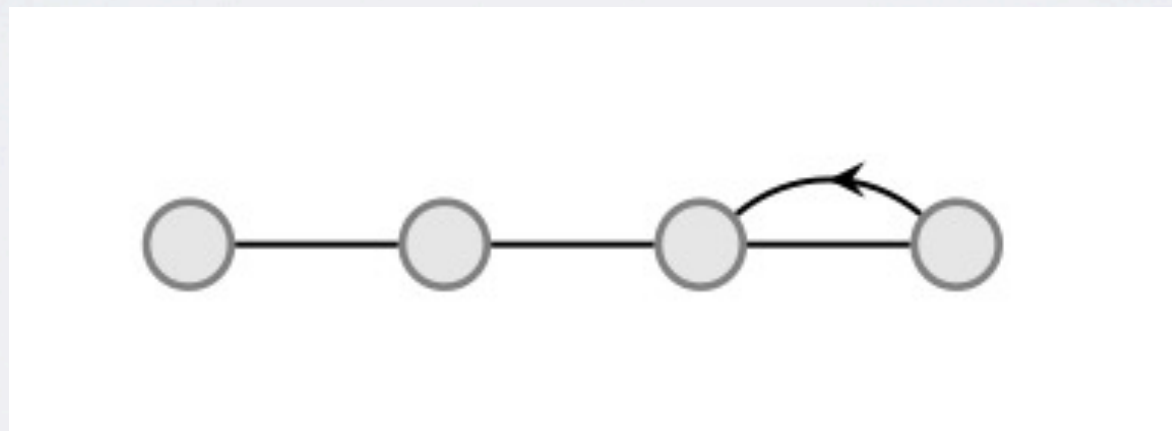
PLANAR MAPS

- A *planar map* is a graph drawn in the plane without edges crossing



ROOTED AND NON-SEPARABLE

- A planar map is *rooted* by orienting an edge

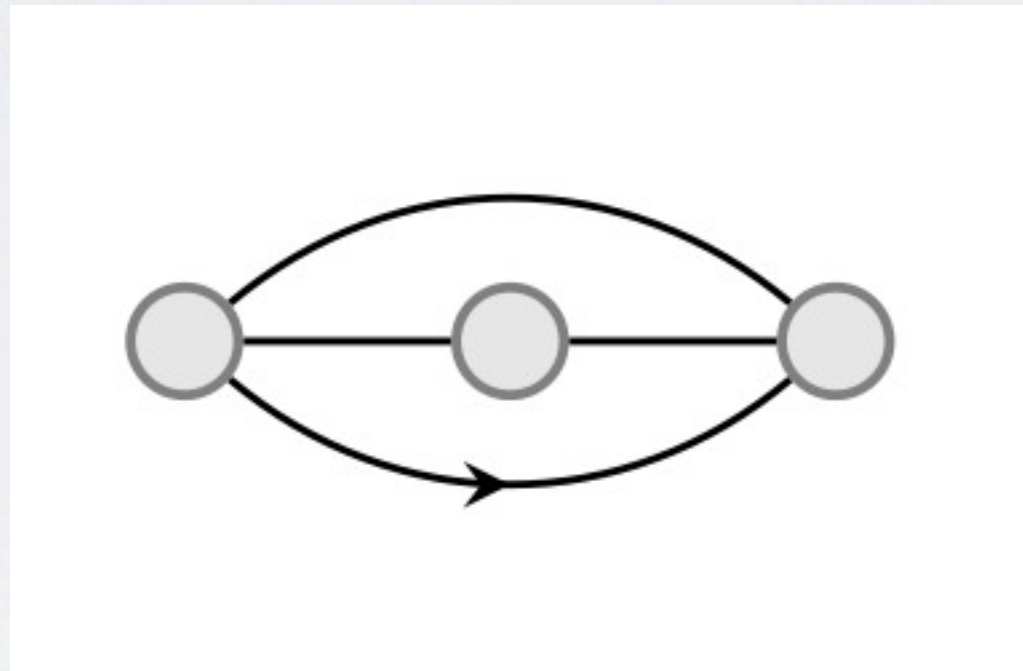


separable

- A planar map is *separable* if it falls a part when we remove a vertex

ROOTED AND NON-SEPARABLE

- A planar map is *rooted* by orienting an edge

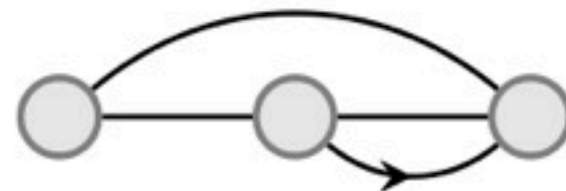
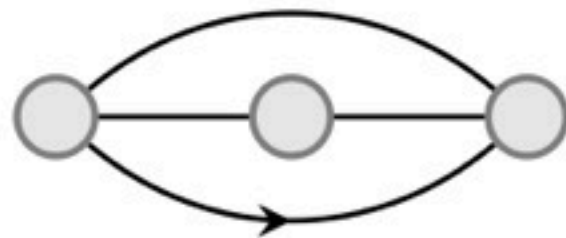
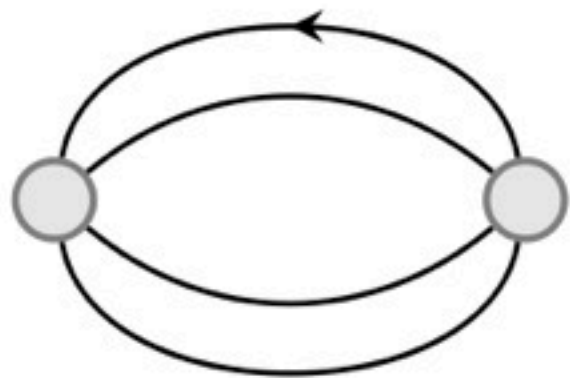


non-separable

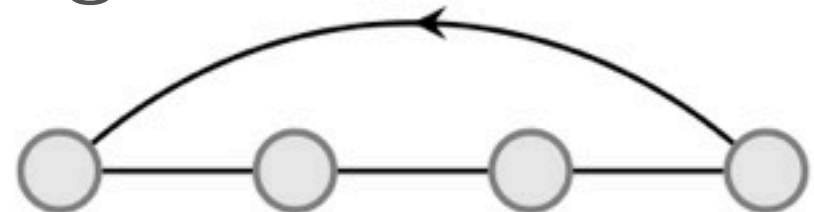
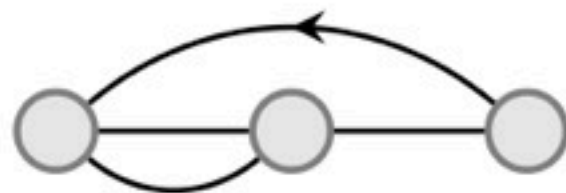
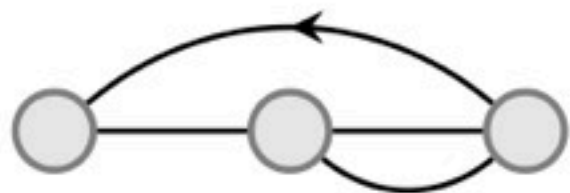
- A planar map is *separable* if it falls a part when we remove a vertex

MAPS

- Non-separable rooted planar maps will be called *maps*



All maps with 4 edges



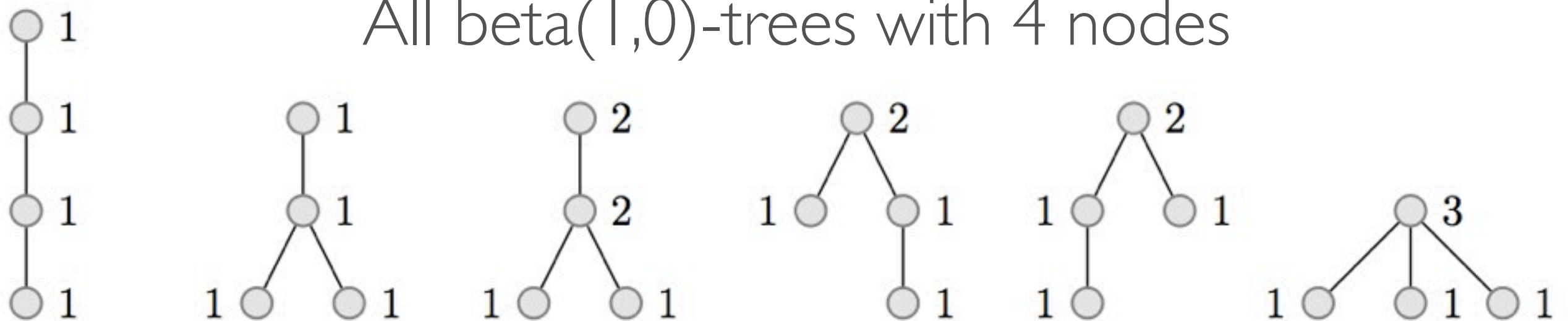
WHY STUDY THESE THINGS

- Equinumerous with permutations that are sortable in two passes through a stack (Goulden & West 1996)
- Appear in statistical physics (Schaeffer & Zinn-Justin 2004)
- Connected to other combinatorial objects

BETA(1,0)-TREES

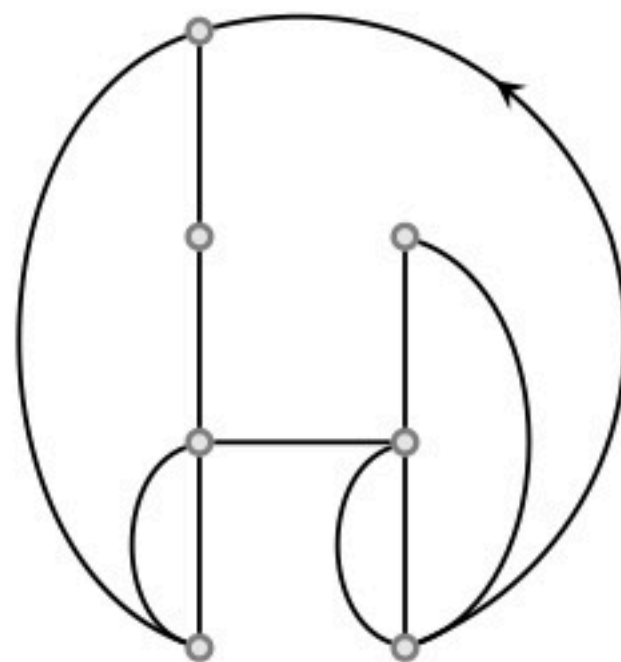
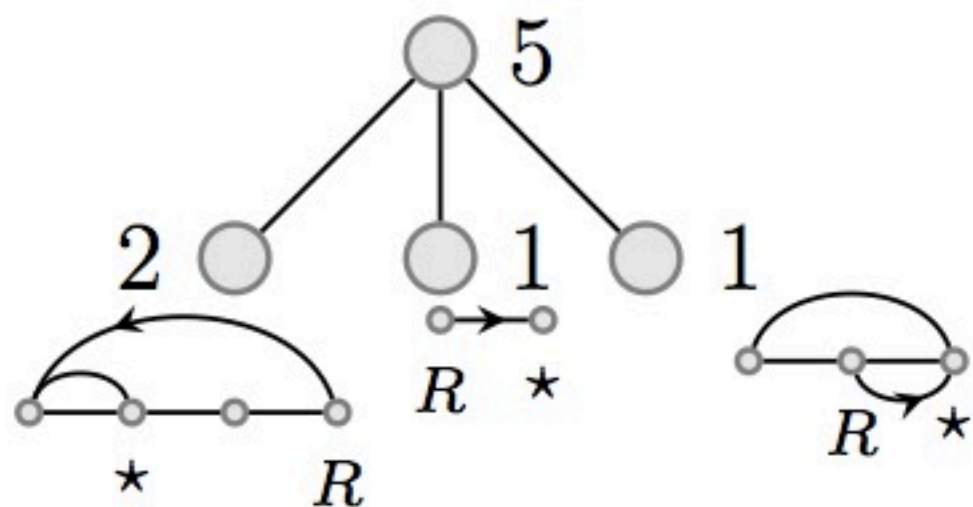
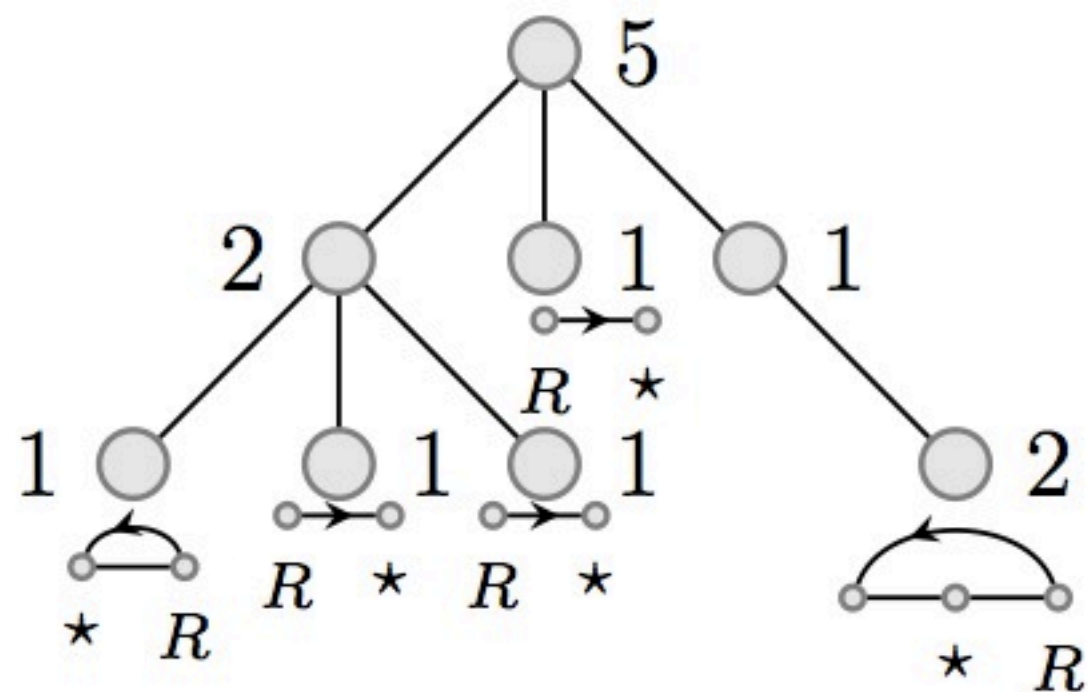
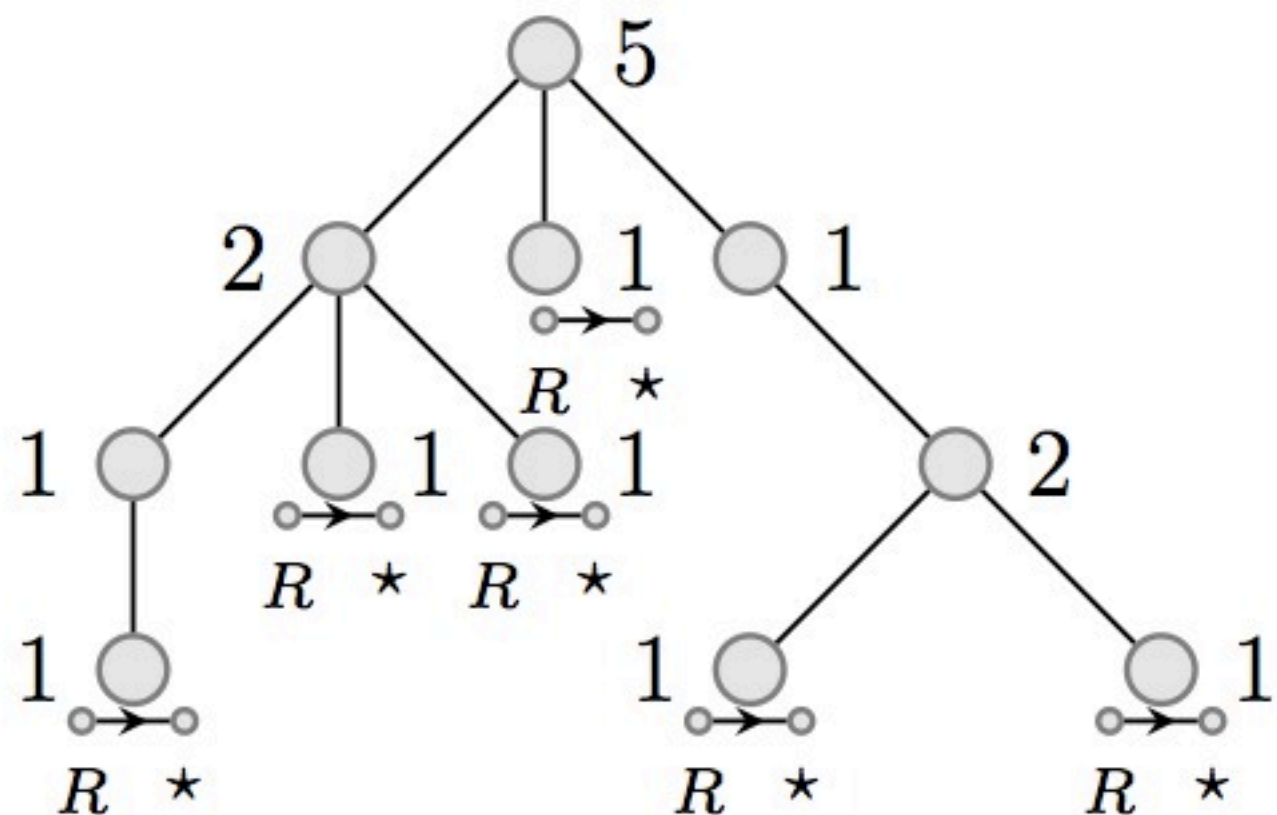
- Cori, Jacquard and Schaeffer (1997) defined beta(1,0)-trees
- Drawn in the plane, marked according to certain rules

All beta(1,0)-trees with 4 nodes



- In bijection with the maps

TREE TO MAP



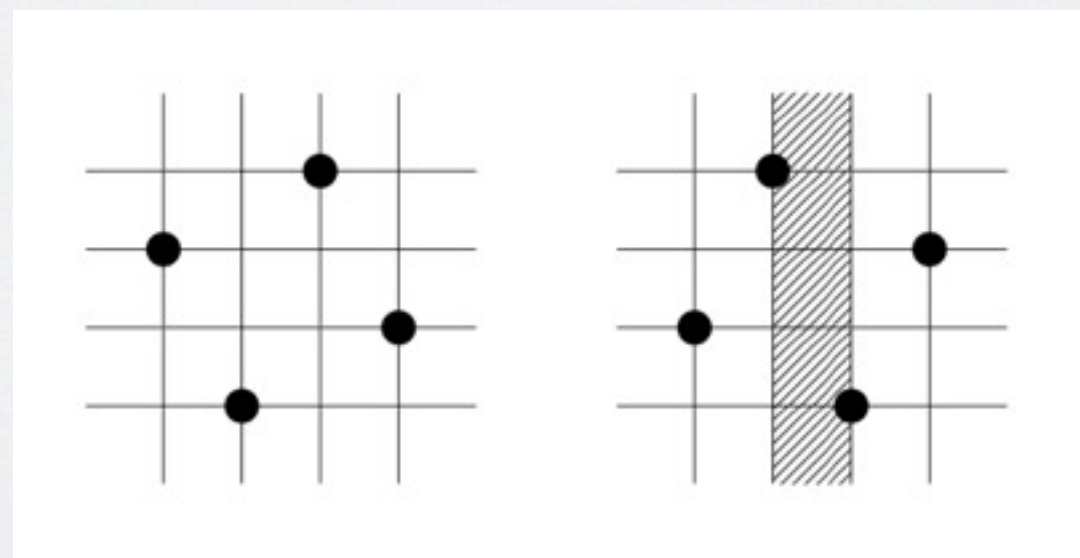
PERMUTATIONS

- A *permutation* is a list of length n where each of the numbers 1 to n appear exactly once
- 5642173 is a permutation of length 7

CONNECTION WITH TREES



- Claesson, Kitaev and Steingrímsson (2009) constructed a bijection between $\text{beta}(1,0)$ -trees and permutations avoiding the patterns

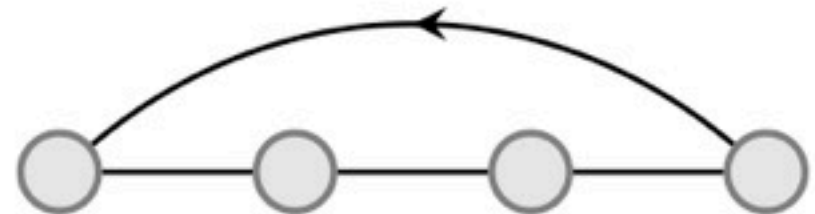
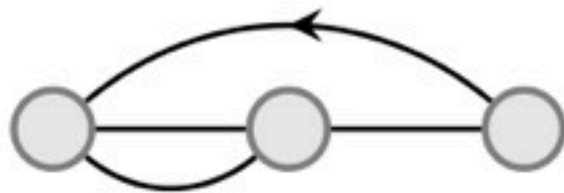
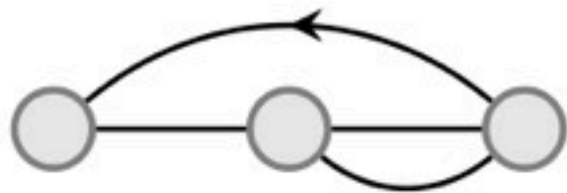
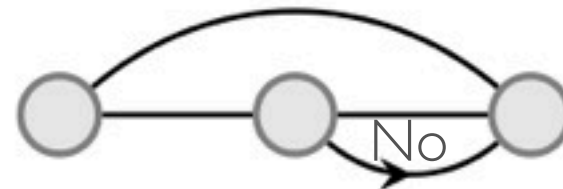
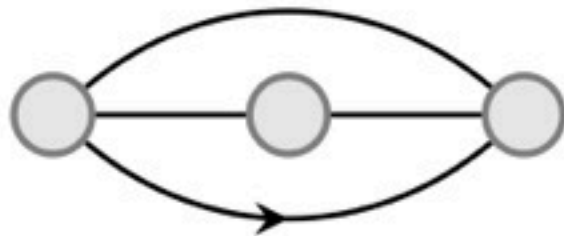
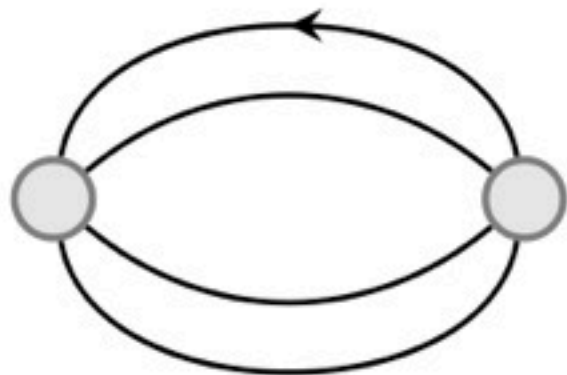


PROPERTIES

- How do properties transfer under the bijections between
 - maps
 - trees
 - permutations

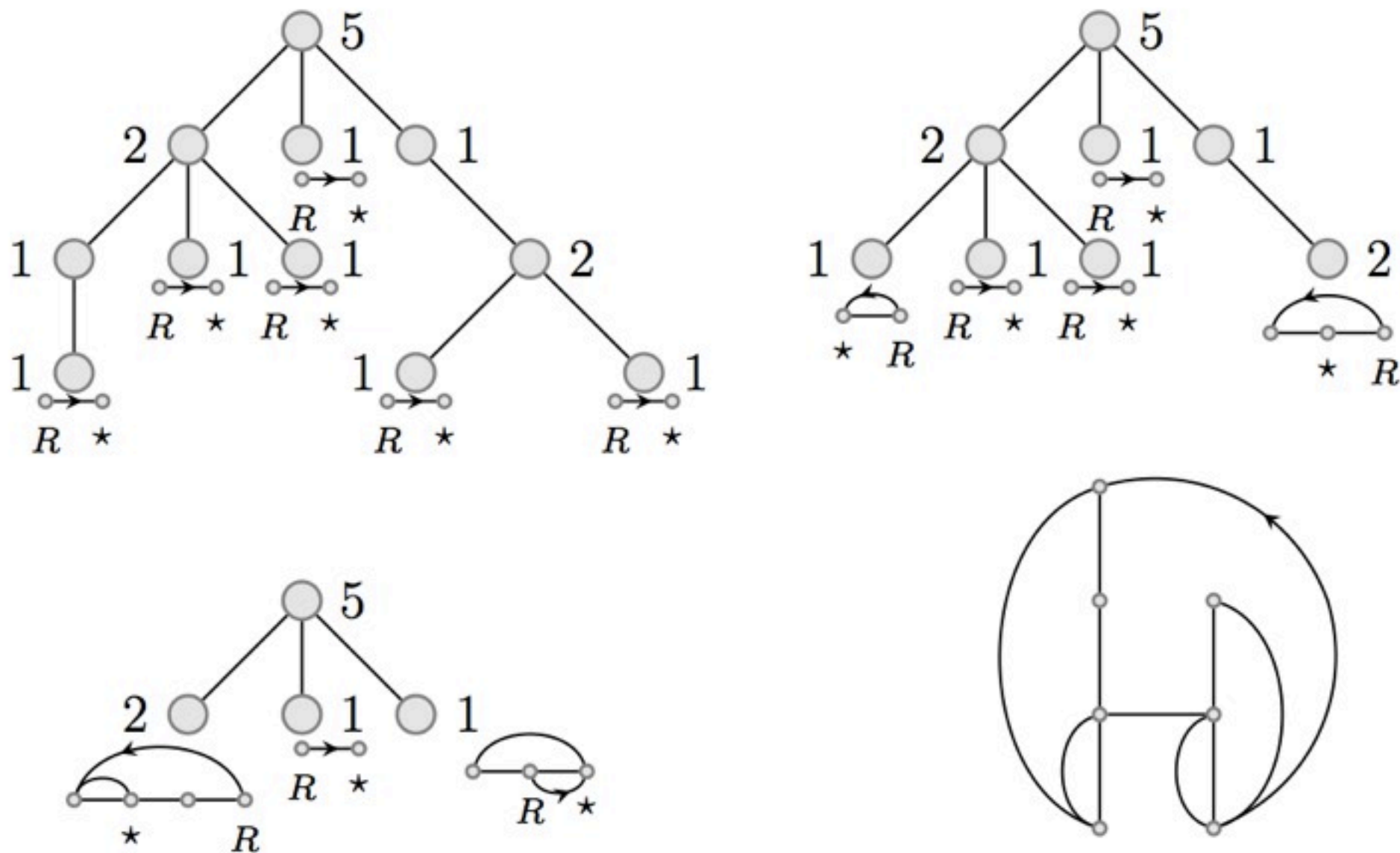
PRIMITIVE MAPS

- A map is *primitive* if it has no internal face with two edges



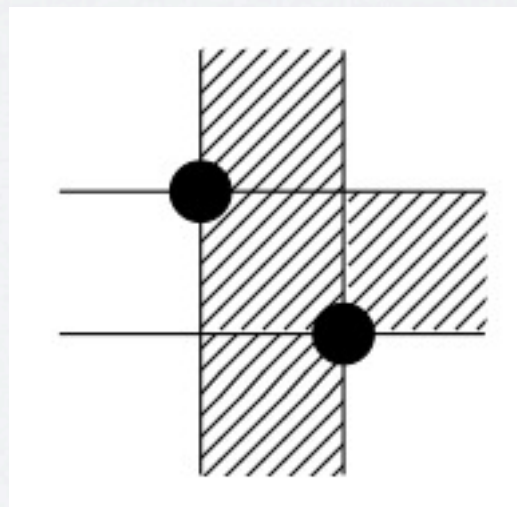
PRIMITIVE TREES

- Theorem (Kitaev, Salimov, Severs, Ú).
- A tree corresponds to a primitive map if and only if it has no vertex which is a single child and has maximal label



PRIMITIVE PERMUTATIONS

- Theorem (Kitaev, Salimov, Severs, Ú).
A permutation corresponds to a primitive map if and only if it avoids the mesh pattern



Questions?