Pattern Avoidance and Non-Crossing Subgraphs of Polygons

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We illustrate a connection between independent sets arising from the study of permutations avoiding the pattern 132 and the classical problem of enumerating certain non-crossing subgraphs. The methods developed can be extended to enumerate subclasses of permutations avoiding the pattern 1324, the only principal permutation class of length 4 that remains unenumerated.

More precisely, an independent set of k vertices in a graph we define and a sequence of positive integers of length k determine a unique permutation avoiding 132. Instead of enumerating the independent sets directly we show they are in bijection with the non-crossing subgraphs in a complete graph drawn on a regular polygon. Enumerating these non-crossing subgraphs is a classical problem, see e.g. Comtet [1]. For our purposes we use the generating function found by Flajolet and Noy [2]:

$$F(x,y) = 1 + x \cdot F(x,y) + \frac{xy \cdot F(x,y)^2}{1 - y \cdot (F(x,y) - 1)}.$$

Here x marks the left-to-right-minima of the permutation and y marks vertices in the graph (or edges in the polygon). Setting $y = y/(1-y)$, $x = xy$, and collecting by powers of y gives the Narayana triangle enumerating the permutations avoiding 132 by their number of left-to-right-minima.

Using the same techniques as above we show that there is a generalized polygon whose edges correspond to the vertices in the graph and enumerate certain subclasses of permutations avoiding 1324.

References.

[1] L. Comtet, Advanced combinatorics, D. Reidel Publishing Co., Dordrecht (1974).
[2] P. Flajolet and M. Noy, Analytic combinatorics of non-crossing configurations,*Discrete Math.* **204** (1999) 203-229.

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