# Pattern Avoidance and Non-Crossing Subgraphs of Polygons

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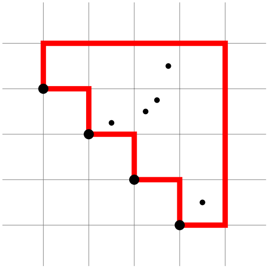
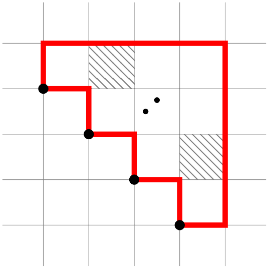
Permutation Patterns, 2015

# 132-avoiding permutations

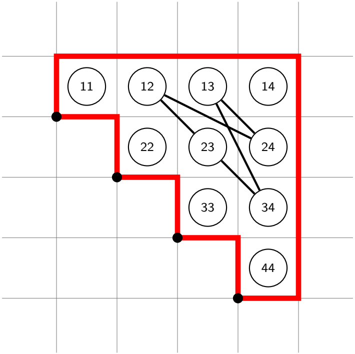
Given any permutation, $\pi$, we can extract the left-to-right minima.



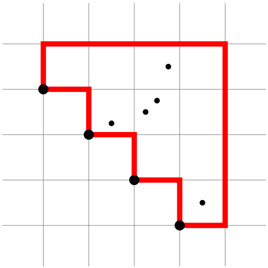
For any permutation $\pi \in Av_n(132)$



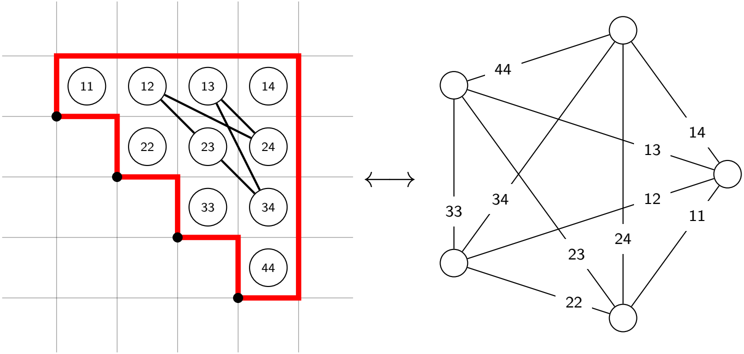
Given this representation we can construct a graph



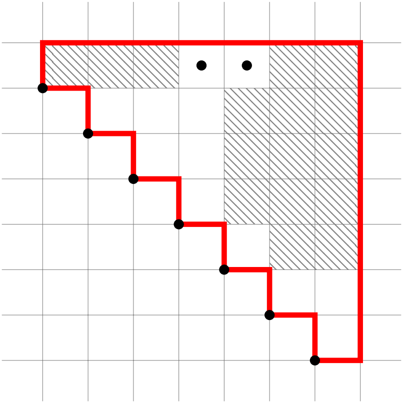
An independent set of size $k$ and a positive integer sequence of length $k$ uniquely determines a 132-avoider.

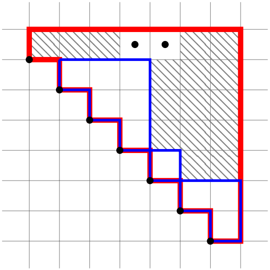


There is in bijection with non-crossing subgraphs on a regular polygon and the independent sets.



We can also directly enumerate this.





| $d$ | $F$ |
|---|---|
| 0 | $x \cdot F$ |
| 1 | $x \cdot y \cdot F^2$ |
| 2 | $x \cdot y^2 \cdot F^2(F-1)$ |
| $\vdots$ | $\vdots$ |
| $n$ | $x \cdot y^n \cdot F^2(F-1)^{n-1}$ |
| $\vdots$ | $\vdots$ |

Deriving the generating function.

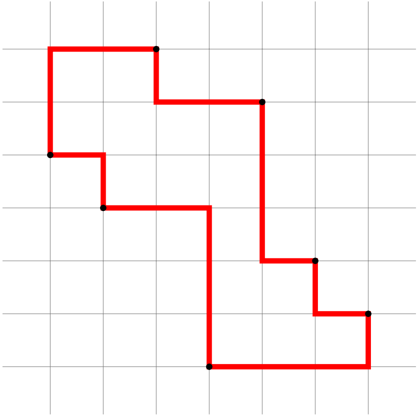
This leads to the following generating function:

$$F(x,y) = 1 + x \cdot F(x,y) + \frac{xy \cdot F(x,y)^2}{1 - y \cdot (F(x,y) - 1)}.$$

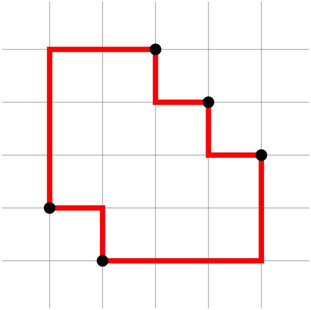
Evaluating $F\left(x, \frac{x}{1-x}\right)$ gives the Catalan numbers.

# 1324-avoiders

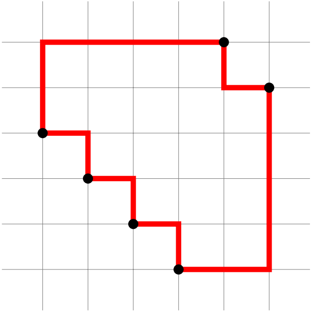
Given $\pi \in Av_n(1324)$ we can extract the boundary.



Non-intersecting boundary of a 1324-avoider.



For 1324-avoiders with non-intersecting boundary and two right to left maxima.



Let:

$$G = \frac{x^2 \cdot F}{1 - y \cdot (F - 1)}$$

Then:

$$H = \frac{G + 1}{1 - y \cdot G} - 1$$